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MA222 - Computational Linear Algebra
Problem Sheet - 4

Vectorization and Re-Use Issues

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1. Consider the matrix product $D = ABC$ where $A \in \mathbb{R}^{m\times r}$, $B \in \mathbb{R}^{r\times n}$ and $C \in \mathbb{R}^{n\times q}$. Assume that all the matrices are stored by column and that the time required to execute a unit-stride saxpy operation of length k is of the form $t(k) = (L + k)\mu$ where L is a constant and μ is the cycle time. Based on this model, when is it more economical to compute D as $D = (AB)C$ instead of as $D = A(BC)$? Assume that all matrix multiplies are done using the *jki*, (gaxpy) algorithm.

2. What is the total time spent in *jki* variant on the saxpy operations assuming that all the matrices are stored by column and that the time required to execute a unit-stride saxpy operation of length k is of the form $t(k) = (L + k)\mu$ where L is a constant and μ is the cycle time? Specialize the algorithm so that it efficiently handles the case when A and B are n-by-n and upper triangular. Does it follow that the triangular implementation is six times faster as the flop count suggests?

3. Give an algorithm for computing $C = A^TBA$ where A and B are n-by-n and B is symmetric. Arrays should be accessed in unit stride fashion within all innermost loops.

4. Suppose $A \in \mathbb{R}^{m\times n}$ is stored by column in $A.col(1 : mn)$. Assume that $m = \ell_1 M$ and $n = \ell_2 N$ and that we regard A as an M-by-N block matrix with ℓ_1-by-ℓ_2 blocks. Given i, j, α, and β that satisfy $1 \le i \le \ell_1$, $1 \le j \le \ell_2$, $1 \le \alpha \le M$, and $1 \le \beta \le N$ determine k so that $A.col(k)$ houses the (i, j) entry of $A_{\alpha\beta}$. Give an algorithm that overwrites $A.col$ with A stored by block an in Figure 1.4.5. How big of a work array is required?

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